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On the cosmology of type IIA compactifications on SU(3)-structure manifolds

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Abstract: We study cosmological properties of type IIA compactifications on orientifolds of SU(3)-structure manifolds with non-vanishing geometric flux. These compactifications give rise to effective 4D $\mathcal{N} = 1$ supergravity theories that do not fall under some recently-proven no-go theorems against de Sitter vacua and slow-roll inflation. Focusing on a well-understood class of models based on coset spaces, however, we can use a refined no-go theorem that rules out de Sitter vacua and slow-roll inflation in all but one case. The refined no-go theorem uses the dilaton and a specific linear combination of the Kähler moduli, which is different from the overall volume modulus. It puts a lower bound on the first slow-roll parameter: $\epsilon \geq 2$. The only case not ruled out is the manifold SU(2)×SU(2), for which we indeed find critical points with ϵ numerically zero. However, all the points we could find have a tachyon corresponding to an eta-parameter $\eta \lesssim -2.4$.

Keywords: Cosmology of Theories beyond the SM, dS vacua in string theory, Flux compactifications

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Contents

1 Introduction

Several beautiful astrophysical measurements over the recent years have provided a fascinating and coherent picture about the evolution and large scale structure of our universe. In particular, we now know that the universe is spatially flat, $|\Omega - 1| \ll 1$, and the latest CMB data from WMAP5 agree with an almost scale-invariant spectrum with scalar spectral index $n_s = 0.96 \pm 0.013$ [1]. These data can be nicely explained by an epoch of cosmic inflation in the early universe [2, 3], modelled by a suitable effective scalar field theory for an inflaton field ϕ, whose positive scalar potential $V(\phi)$ drives the nearly exponential expansion. As the details of the CMB spectrum depend sensitively on the precise shape of the inflaton potential, it is an exciting possibility to try to use CMB measurements as a powerful, and possibly unique, probe of the fundamental theory of matter and gravity

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in the early universe. In order to come closer to this goal, one would like to constrain the huge space of possible effective field theories to a much smaller, more manageable set. This can be done either by improving the available data sets, or by attempts to consistently embed models of inflation in fundamental theories of quantum gravity such as string theory. That the latter is in general nontrivial follows from, e.g., the generic sensitivity of inflation models to even Planck suppressed corrections to the inflaton potential, which cannot be chosen at will in a given UV-complete theory, but take on definite values. This sensitivity concerns in particular the flatness parameters

$$\begin{aligned} \epsilon &\equiv \frac{M_P^2\, g^{AB}\partial_A V \partial_B V}{2\, V^2}\,, \\ \eta &\equiv \text{min eigenvalue}\left(\frac{M_P^2\, \nabla^A \partial_B V}{V}\right)\,, \end{aligned} \tag{1.1}$$

where we have displayed the general expressions for several scalar fields with g^{AB} being the inverse scalar field metric. The self-consistency of the standard slow-roll approximation (and relations such as $n_s = 1 - 6\epsilon + 2\eta$) requires these parameters to be small,

$$\epsilon \ll 1\,, \qquad |\eta| \ll 1\,. \tag{1.2}$$

Another important cosmological observation of the past decade is that also the present universe is in a state of accelerated expansion [4], apparently driven by a non-vanishing vacuum energy with an equation of state very close to that of a small and positive cosmological constant Λ. In an effective field theory setup, an asymptotic de Sitter (dS) phase induced by a constant vacuum energy would correspond to a local minimum of the potential with $\epsilon = 0$ and $\eta > 0$ at $V > 0$.

The moduli of string compactifications are often considered as natural inflaton candidates, and various inflation models have been proposed based on this idea (for recent reviews on the subject, see, e.g., [5] and references therein). These models can roughly be divided into closed string inflation models, in which the inflaton is a closed string modulus, and open string (or brane-) inflation models, where the role of the inflaton is played by a scalar describing some relative brane distance or orientation.[1] In any such model, it is important to stabilize the orthogonal moduli, which one does not want to participate in inflation, in particular if these orthogonal moduli correspond to potentially steep directions of the scalar potential.

In contrast to type IIB string theory, where cosmological model building and moduli stabilization are already quite advanced subjects (following the work of [7–9]), comparatively little is known in type IIA string theory, even though type IIA models are very interesting for several good reasons:

First of all, type IIA orientifolds with intersecting D6-branes (see e.g. [10, 11] for reviews and many more references) offer good prospects for deriving the Standard Model

[1] Mixtures of open and closed string moduli have also been considered as inflaton candidates, e.g., in some variations of D3/D7-brane inflation [6].

from strings, as was recently demonstrated in [12].[2] So, if cosmological aspects can likewise be modelled, one may study questions such as, e.g., reheating much more explicitly.

Second, in type IIA compactifications, all geometrical moduli can be already stabilized at the classical level by fluxes, albeit to AdS vacua in four dimensions [15–18]. The advantage of such models is their explicitness and the possibility to stabilize the moduli in a well-controlled regime (corresponding to large volume and small string coupling) with power law parametric control (instead of logarithmic as in type IIB constructions along the lines of [7]).

The main problem of such type IIA compactifications is that there exist already quite strong no-go theorems against dS vacua and slow-roll inflation: extending the earlier work [19], the authors of [20] prove a no-go theorem against small ϵ in type IIA compactifications on Calabi-Yau manifolds with standard RR and NSNS-fluxes, D6-branes and O6-planes at large volume and with small string coupling. This no-go theorem uses the particular functional dependence of the corresponding scalar potentials on the volume modulus ρ and the 4D dilaton τ. More precisely, using the (ρ, τ)-dependence, they show that the slow-roll parameter ϵ is at least $\frac{27}{13}$ whenever the potential is positive, ruling out slow-roll inflation in a near-dS regime, as well as meta-stable dS vacua. As was already emphasized in [20], however, the inclusion of other ingredients such as NS5-branes, geometric fluxes and/or non-geometric fluxes evade the assumptions that underly this no-go theorem. From these ingredients, especially geometric fluxes are quite natural in a type IIA context, as it is known that D6-branes and fluxes in that case have a stronger backreaction than the ISD three-form fluxes in IIB [9], deforming the internal geometry away from a Calabi-Yau manifold. Following the mathematical work of [21] there is now also a better understanding of the resulting SU(3)-structure manifolds and their application to compactifications with fluxes, see e.g. [22–30]. In [31], a combination of geometric fluxes, KK5-branes and discrete Wilson lines was indeed argued to allow for dS minima. These ingredients were used in [32] to construct large field inflationary models with very interesting experimental predictions.

In the recent work [33], F_0 flux (i.e. non-vanishing Romans mass) and geometric flux were identified as "minimal" additional ingredients in order to circumvent the no-go theorem of [20]. In the present paper, we discuss the question to what extent the recently constructed type IIA $\mathcal{N} = 1$ AdS_4 vacua with SU(3)-structure [34–40] can be used for inflation or dS vacua. In particular, the coset models with SU(3)-structure could be candidates for circumventing the no-go theorem of [20], as they all have geometric fluxes and allow for non-vanishing Romans mass. Specifically, we investigate whether the scalar potentials in the closed string moduli sector that were already investigated in [40] can be flat enough in order to allow for inflation by one of the closed string moduli. For this to be the case the parameter ϵ must be small enough in some region of the positive closed string scalar potential. In addition, this analysis is also relevant for open string inflation in these IIA vacua, since in this case we have to find closed string minima of the scalar potential, i.e. $\epsilon = 0$ somewhere in the closed string moduli space. Having a point with $\epsilon = 0$ would also be a necessary condition for a dS vacuum somewhere in moduli space.

[2] On the other hand, for recent progress in GUT model building in type IIB orientifolds see [13]. Furthermore, there has recently been a lot of activity in model building in F-theory following the work of [14].

For finding a small ϵ, however, it is not relevant whether the effective field theory actually has a supersymmetric vacuum. Because of this we also extend our analysis to two more coset spaces that allow for an SU(3)-structure but do not admit a supersymmetric AdS vacuum.

The main result of our investigation is that we can apply, for all but one model, a refined no-go theorem of [41] that does *not* just use the volume modulus and the dilaton, but also some of the other Kähler moduli.[3] These would not have been ruled out by the no-go theorem of [20] (except for the example of positive curvature, which we already excluded in [40]). Just as in [20], it is the epsilon parameter, i.e., first derivatives of the potential that cannot be made small. Our results in particular show that it is important to make sure that the potential has a critical point (or small first derivative) in *all* directions in moduli space. Moreover, the refined no-go theorem, just as the one of [20], is of a different nature than the no-go theorems developed in [42], which assume a vanishing (or small) first derivative and then show that, under certain conditions, the eta parameter cannot be made small enough.

The coset model we do not rule out by a no-go theorem corresponds to the group manifold SU(2)×SU(2). For this model, we can indeed show that points in moduli space with $\epsilon \approx 0$ exist. The points we could find, however, have a tachyonic direction corresponding to $\eta \lesssim -2.4$, making them useless for our intended phenomenological applications. We could not rule out the existence of analogous points with smaller $|\eta|$.

The organization of the paper is as follows: in section 2, we review the no-go theorems discussed in [20] and [33], paying particular attention to the role of geometric fluxes. In section 3, we discuss a refined no-go theorem of [41] that can be applied to SU(3)-structure compactifications with a certain form of the triple intersection numbers. We also comment on the possibility of circumventing these no-go theorems by adding a number of additional ingredients. In section 4, we apply the no-go theorem of section 3 to the coset spaces and group manifolds with SU(3)-structure discussed in [40] and the two extra non-supersymmetric cosets, taken from [39]. Our results are summarized in section 5. Some technical details on the moduli space are relegated to appendix A.

2 Geometric fluxes and no-go theorems in the volume-dilaton plane

We start by reviewing previously derived no-go theorems [20] (see also [31, 33]) that exclude slow-roll inflation and dS vacua in the simplest compactifications of massive type IIA supergravity, focusing in particular on the role played by the curvature of the internal space.

Classically, the four-dimensional scalar potentials of such compactifications may receive contributions from the NSNS H_3-flux, geometric fluxes,[4] O6/D6-branes and the RR-fluxes[5]

[3] Problems with field directions orthogonal to the (ρ, τ)-plane were also discussed in [33], where attempts were made to construct dS vacua on manifolds that are products of certain three-manifolds.

[4] Geometric flux is not a terribly well-defined concept. For us the internal manifold will have geometric flux if the Ricci scalar R is non-zero. In the special case of group and coset manifolds, the geometric flux can be related to the structure constants $f^{\alpha}{}_{\beta\gamma}$.

[5] We use the democratic formalism for the RR-fluxes [43] introducing extra fluxes for $p = 6, 8, 10$ and a duality constraint. Next, we impose the compactification ansatz $F_p^{\text{tot}} = F_p + \text{dvol}_4 \wedge \tilde{F}_{p-4}$, where F_p and

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F_p, $p = 0, 2, 4, 6$ leading to, respectively, the following terms:

$$V = V_3 + V_f + V_{O6/D6} + V_0 + V_2 + V_4 + V_6, \tag{2.1}$$

where $V_3, V_0, V_2, V_4, V_6 \geq 0$, and V_f and $V_{O6/D6}$ can a priori have either sign.

In [20] the authors studied the dependence of this scalar potential on the volume modulus $\rho = (\mathrm{Vol})^{1/3}$ and the four-dimensional dilaton $\tau = e^{-\phi}\sqrt{\mathrm{Vol}}$. Using only this (ρ, τ)-dependence, they could derive a no-go theorem in the absence of metric fluxes that puts a lower bound on the first slow-roll parameter,

$$\epsilon \equiv \frac{K^{A\bar{B}}\partial_A V \partial_{\bar{B}} V}{V^2} \geq \frac{27}{13}, \quad \text{whenever } V > 0, \tag{2.2}$$

where $K^{A\bar{B}}$ denotes the inverse Kähler metric, and the indices $A, B, \ldots$ run over all moduli fields. This then not only excludes slow-roll inflation but also dS vacua (corresponding to $\epsilon = 0$).

The lower bound (2.2) follows from the observation that a linear combination of the derivatives with respect to ρ and τ is always greater than a certain positive multiple of the scalar potential V. More precisely, it is not difficult to obtain the general scaling behavior of these terms with respect to ρ and τ,

$$V_3 \propto \rho^{-3}\tau^{-2}, \qquad V_p \propto \rho^{3-p}\tau^{-4}, \qquad V_{O6/D6} \propto \tau^{-3}, \qquad V_f \propto \rho^{-1}\tau^{-2}, \tag{2.3}$$

which then implies for the scalar potential (2.1)

$$-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + \sum_{p=2,4,6} pV_p - 2V_f. \tag{2.4}$$

Hence, whenever the contribution from the metric fluxes V_f is zero or negative, the right hand side in (2.4) is at least equal to $9V$, which can then be translated to the above-mentioned lower bound $\epsilon \geq \frac{27}{13}$. Avoiding this no-go theorem without introducing any new ingredients would thus require $V_f > 0$. Since $V_f \propto -R$, where R denotes the internal scalar curvature, this is equivalent to demanding that the internal space has negative curvature. Since all terms in V scale with a negative power of τ we see from (2.1) and (2.3) that we then also need $V_{O6/D6} < 0$ to avoid a runaway.

Following a related argument in [33], one can identify another combination of derivatives with respect to ρ and τ that sets a bound for ϵ:

$$-3\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + 6V_3 - 6V_0 + 6V_4 + 12V_6 \geq 9V - 6V_0. \tag{2.5}$$

In the case of vanishing mass parameter, we have $V_0 = 0$, and (2.5) implies $\epsilon \geq \frac{9}{7}$. Therefore, we need to have $V_f > 0$, $V_{O6/D6} < 0$ and $V_0 \neq 0$ in order to avoid the above no-go theorems. Note that the only real restriction here is that we have to have a compact space with

$\tilde{F}_{p-4}$ have only internal indices. The duality condition allows then to express $\tilde{F}_{6-p}$ in terms of F_p with $p = 0, 2, 4, 6$.

negative curvature since we are always free to turn on F_0-flux and to do an orientifold projection. Of the coset models discussed in [40] only the $\frac{G_2}{SU(3)}$ case has positive curvature over the whole parameter space and is thus ruled out by the above no-go theorem. The other coset spaces studied in [40], on the other hand, do allow for negative curvature and are thus not affected by the no-go theorem of [20]. This was already noted in [40] and justifies a closer look at these models. As already mentioned in the introduction we will also discuss two more coset spaces not analyzed in [40], which do not admit a supersymmetric AdS vacuum.

3 SU(3)-structure manifolds and a different no-go theorem

The no-go theorems described in the previous section are very general in the sense that they do not assume anything specific about the geometry of the internal manifold apart from the possible presence of geometric fluxes, p-form fluxes and O6/D6-brane sources. An obvious way around the no-go theorem is to look at internal spaces with geometric fluxes leading to negative curvature. In this section we will use the technology of SU(3)-structures and discuss a refined no-go theorem [41] that holds for an important subclass of compactifications with geometric fluxes.

On manifolds with SU(3)-structure, the structure group of the tangent bundle can be reduced to SU(3), which implies the existence of a non-vanishing, globally defined spinor field η_+.[6] In type II compactifications, the effective 4D theory can then be described by an $\mathcal{N} = 2$ supergravity Lagrangian, which, under suitable orientifold projections, turns into an effective $\mathcal{N} = 1$ theory. From bilinears of the spinor field one can form a globally defined real two-form, J, and a complex decomposable three-form, Ω, which generalize, respectively, the Kähler form and the holomorphic three-form of a Calabi-Yau manifold:[7]

$$J_{mn} = i\eta_+^\dagger \gamma_{mn}\eta_+, \qquad \Omega_{mnp} = \eta_-^\dagger \gamma_{mnp}\eta_+, \tag{3.1}$$

where η_- is the complex conjugate of η_+, and γ_{mn}, etc. denote the weighted antisymmetrized products of (internal) gamma matrices. Fierz identities then imply

$$\Omega \wedge J = 0\,, \tag{3.2}$$

$$\Omega \wedge \Omega^* = \frac{4i}{3} J^3 \neq 0, \tag{3.3}$$

and the 6D volume-form is

$$\mathrm{dvol}_6 = \frac{1}{6} J^3 = -\frac{i}{8}\Omega \wedge \Omega^*\,. \tag{3.4}$$

In our concrete models, we will introduce a configuration of O6-planes such that the resulting low-energy theory in four dimensions is $\mathcal{N} = 1$ supersymmetric. In order not

[6]There are more general ways one can decompose the 10D spinor fields into 6D and 4D spinors such that one still has a 4D $\mathcal{N} = 2$ (or after orientifolding $\mathcal{N} = 1$) supergravity theory. These lead to so-called SU(3)×SU(3)-structure and the low-energy supergravity is discussed in [44–46, 48].

[7]On Calabi-Yau manifolds, the spinor field η_+ is covariantly constant with respect to the Levi-Civita connection, implying $\mathrm{d}J = \mathrm{d}\Omega = 0$. In our case however, we have generically $\mathrm{d}J \neq 0$ and $\mathrm{d}\Omega \neq 0$, and the decomposition of these quantities in terms of SU(3)-representations defines the intrinsic torsion classes [21] (see also [30] for a review). These torsion classes then lead to a non-vanishing curvature scalar and can thus be related to the geometric flux.

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to be projected out, H, F_2 and F_6 should be odd, and F_0 and F_4 should be even under each orientifold involution. Furthermore, the condition that the orientifold projection does not fully break the supersymmetry requires J and $\mathrm{Re}\Omega$ to be odd, and $\mathrm{Im}\Omega$ to be even under each orientifold involution. The holomorphic variables of the low-energy $\mathcal{N} = 1$ theory sit then in the expansion of the complex combinations $J_c = J - i\delta B$ and $\Omega_c = e^{-\Phi}\mathrm{Im}\Omega + i\delta C_3$ [24], where δB and δC_3 are fluctuations around the background of the NSNS two-form and RR three-form potential, respectively, and Φ is the 10D dilaton.

Just as in Calabi-Yau compactifications, one tries to expand the complexified J_c and Ω_c in a suitable basis of forms to yield an analogue of Kähler moduli, k^i, and complex structure moduli $\tilde{u}^I = \tau^{-1}u^I$:[8]

$$J_c = (k^i - ib^i)Y_i^{(2-)} \equiv t^i Y_i^{(2-)}\,, \qquad (i = 1, \ldots, h^{2-})\,, \tag{3.5a}$$

$$\Omega_c = (u^I + ic^I)Y_I^{(3+)} \equiv z^I Y_I^{(3+)}\,, \qquad (I = 1, \ldots, h^{3+})\,. \tag{3.5b}$$

Here, $Y_i^{(2-)}$ and $Y_I^{(3+)}$ are a set of expansion forms with suitable parity under the orientifold involution, as indicated by the superscript $+/-$. In contrast to the Calabi-Yau case, they are in general not harmonic, since J_c and Ω_c are not necessarily closed. The numbers h^{2-} and h^{3+} therefore do not count harmonic forms and should not be confused with the Hodge numbers. The easiest way to satisfy the compatibility condition (3.2) is to impose it for all choices of the moduli t^i and z^I, which implies for our basis forms

$$Y_i^{(2-)} \wedge Y_I^{(3\pm)} = 0\,, \tag{3.6}$$

for all i and I.

As the expansion forms are in general not harmonic, the corresponding 4D scalars t^i, z^I are usually not massless. There is thus no clear separation into massive and massless modes as in conventional Calabi-Yau compactifications, and the identification of a suitable expansion basis is generally an important open problem. One approach is to derive constraints on the basis from the requirement of obtaining an effective supergravity Lagrangian with an appropriate amount of supersymmetry ($\mathcal{N} = 2$, or $\mathcal{N} = 1$ after orientifolding). Another would be to find a consistent truncation, and a third, perhaps most physical, would be to try to establish a hierarchy of masses and keep only the light fields. See [23–26, 44] for various approaches to this problem. The position adopted in [40] is that on group manifolds or coset spaces, at least, a natural expansion basis is provided by the left-invariant forms. In our concrete examples we will restrict our discussions to these cases and hence only consider SU(3)-structures that are likewise left-invariant. As expansion forms we will then choose for $Y_i^{(2-)}$ the left-invariant odd two-forms and for $Y_I^{(3+)}$ the left-invariant even three-forms. In the concrete models of section 4 there are either no odd left-invariant five forms or we will arrange for them to be projected out by orientifolding, so that the extra condition (3.6) is trivial.

[8] Note that there are h^{3+} moduli z^I whose real parts $\mathrm{Re}\,z^I = u^I$ depend on τ and the $(h^{3+} - 1)$ complex structure moduli. We can take out the τ-dependence and define $u^I = \tau\tilde{u}^I$. So strictly speaking the $\tilde{u}^I, I = 1, \ldots, h^{3+}$ are not the complex structure moduli but rather functions of the $(h^{3+} - 1)$ complex structure moduli that are not all independent.

3.1 The scalar potential in SU(3)-structure compactifications

The main idea of the no-go theorems discussed in section 2 is to find a subset of the moduli along which the first derivatives of the scalar potential cannot be simultaneously made sufficiently small for slow-roll inflation. In section 2, the relevant subset of the moduli consists of the overall volume modulus ρ and the 4D dilaton τ.

We will now closely follow [41] and discuss another no-go theorem that is very similar in spirit, but concerns a different two-dimensional slice in moduli space that no longer involves the overall volume ρ, but a different (albeit related) Kähler modulus. In order to write down the dependence of the scalar potential on the moduli, we first define the Kähler potential [24][9]

$$K = K_k + K_c + 3\ln(8\kappa_{10}^2 M_P^2)\,, \tag{3.7}$$

where K_k and K_c are the parts containing, respectively, the Kähler and complex structure/dilaton moduli. They are given by

$$K_k = -\ln\int_M \frac{4}{3}J^3 = -\ln\left(8\,\mathrm{Vol}\right)\,, \tag{3.8a}$$

$$K_c = -2\ln\int_M 2\,e^{-\Phi}\mathrm{Im}\,\Omega\wedge e^{-\Phi}\mathrm{Re}\,\Omega = -4\ln\tau + \tilde{K}_c\,, \tag{3.8b}$$

where $\tilde{K}_c$ only depends on the complex structure moduli $\tilde{u}^I$ of footnote 8. Also in the second line, $e^{-\Phi}\mathrm{Re}\,\Omega$ should be considered as a function of $e^{-\Phi}\mathrm{Im}\,\Omega$.[10] Furthermore, from the relation (3.4) we find

$$\mathrm{Vol} = \frac{e^{-K_k}}{8} = \frac{\kappa_{ijk}k^i k^j k^k}{6}\,, \tag{3.9}$$

where κ_{ijk} denotes the triple intersection number, given in terms of the odd two-forms $Y_i^{(2-)}$ as

$$\kappa_{ijk} = \int_M Y_i^{(2-)}\wedge Y_j^{(2-)}\wedge Y_k^{(2-)}\,. \tag{3.10}$$

The Kähler metric is given in the standard way in terms of the Kähler potential[11]

$$K_{I\bar{J}} = \frac{\partial^2 K_k}{\partial z^I \partial \bar{z}^{\bar{J}}}\,, \qquad K_{i\bar{\jmath}} = \frac{\partial^2 K_c}{\partial t^i \partial \bar{t}^{\bar{\jmath}}}\,. \tag{3.11}$$

It is also convenient to introduce a rescaled inverse Kähler metric for the complex structure sector $\widehat{K}^{I\bar{J}} = \tau^{-2}K^{I\bar{J}}$ so that $\widehat{K}^{I\bar{J}}$ is independent of τ.

For SU(3)-structure manifolds, geometric flux describes the non-closure of the forms J and Ω, i.e. the non-vanishing intrinsic torsion, and, following our expansion (3.5), we parameterize them in terms of matrices r_{iI} defined by [27]

$$\mathrm{d}Y_i^{(2-)} = r_{iI}Y^{(3-)I}\,. \tag{3.12}$$

[9] The constant last term makes e^K dimensionless.

[10] Indeed, $e^{-\Phi}\Omega$ should be a decomposable form, which, according to [47], implies that one can find $e^{-\Phi}\mathrm{Re}\,\Omega$ from $e^{-\Phi}\mathrm{Im}\,\Omega$.

[11] We will use the indices $I, J, \ldots$ for both the real coordinates u^I as well as the complex coordinates z^I. There should be no confusion as the field itself is always indicated. In the definition of the Kähler metric the bar-notation emphasizes that the second derivatives is with respect to the complex conjugate. Likewise for the indices $i, j, \ldots$.

In terms of the above quantities, the scalar potential coming from the $N = 1$ supergravity superpotential [24, 41],[12] splits into a sum of the following terms that can be attributed to, respectively, the H_3 flux, the metric flux, the $O6/D6$-branes and the different RR p-form fluxes[13]

$$
\begin{aligned}
V_3 =& \frac{\widehat{K}^{I\bar{J}} a_I a_J}{\tau^2 \,\mathrm{Vol}}\,, \\
V_f =& \frac{1}{2\tau^2 \,\mathrm{Vol}} \left(\widehat{K}^{I\bar{J}} r_{iI} r_{jJ} k^i k^j - 2(u^I r_{iI} k^i)^2 - 4\mathrm{Vol}(\kappa_i k^i)^{-1\,jk} r_{jI} r_{kJ} u^I u^J \right)\,, \\
V_{O6/D6} =& -\frac{u^I b_I}{\tau^3}\,, \\
V_0 =& \frac{c_0 \mathrm{Vol}}{\tau^4}\,, \\
V_2 =& \frac{c_2^i\, c_2^j}{\tau^4 \,\mathrm{Vol}} \left(\kappa_{ikl} k^k k^l\, \kappa_{jmn} k^m k^n - 4\,\mathrm{Vol}\, \kappa_{ijn} k^n \right) = \frac{16\, c_2^i\, c_2^j \,\mathrm{Vol}\, K_{i\bar{j}}}{\tau^4}\,, \\
V_4 =& \frac{c_{4i}\, c_{4j}}{\tau^4 \,\mathrm{Vol}} \left(2k^i k^j - 2\mathrm{Vol}(\kappa_k k^k)^{-1\,ij} \right) = \frac{c_{4i}\, c_{4j} K^{i\bar{j}}}{2\,\tau^4 \,\mathrm{Vol}}\,, \\
V_6 =& \frac{c_6}{\tau^4 \,\mathrm{Vol}}\,.
\end{aligned}
\tag{3.13}
$$

Here, $(\kappa_k k^k)^{-1\,ij}$ denotes the inverse matrix of $\kappa_{kij} k^k$, and the coefficients a_I, b_I, c_2^i, c_{4i} and $c_0, c_6 \geq 0$ depend on the fluxes, $O6/D6$-brane charges and the axion moduli.[14] As mentioned before, the only contributions to the scalar potential that are not necessarily positive are V_f and $V_{O6/D6}$, and we need V_f to be positive in order to evade the no-go theorem of [20] and $V_{O6/D6} < 0$ to avoid a runaway in the τ direction.

3.2 Special intersection numbers

The coset examples of SU(3)-structure manifolds discussed in [38–40] have special intersection numbers that allow a split of the index i of the Kähler moduli into $\{0, a\}, a = 1, \ldots, (h^{2-} - 1)$, such that the only non-vanishing intersection numbers are

$$\kappa_{0ab} \equiv X_{ab}\,. \tag{3.14}$$

We now introduce a variable similar to ρ above by defining

$$k^a = \sigma \chi^a\,, \tag{3.15}$$

where σ is the overall scale of $(h^{2-} - 1)$ Kähler moduli and the χ^a are constrained by $X_{ab}\chi^a\chi^b = 2$. The volume can now simply be written as $\mathrm{Vol} = k^0\sigma^2$. We can then write V_2 and V_4 in terms of X_{ab} rather than κ_{ijk} and spell out the explicit dependence on k^0 and σ:

$$
\begin{aligned}
V_2 &= \frac{4}{\tau^4 k^0 \sigma^2} \left(c_2^0 c_2^0 \sigma^4 + c_2^a c_2^b (k^0)^2 \sigma^2 (X_{ac}\chi^c X_{bd}\chi^d - X_{ab}) \right)\,, \\
V_4 &= \frac{2}{\tau^4 k^0 \sigma^2} \left(c_{40} c_{40} (k^0)^2 + c_{4a} c_{4b} \sigma^2 (\chi^a \chi^b - X^{-1ab}) \right)\,.
\end{aligned}
\tag{3.16}
$$

[12] See also [48] for a derivation of the scalar potential in the general SU(3)×SU(3)-case.

[13] Since the explicit models we will consider later have $h^{2+} = 0$, there are no gauge fields arising from C_3, and we need not consider D-term contributions (which might in general arise in the presence of metric fluxes [27, 28]).

[14] We refer the interested reader to [41] for the explicit form of these coefficients.

It follows from the positivity of the Kähler metric (cf. the corresponding terms in (3.13)) and the orthogonality of k^0 and k^a that the two terms in V_2 and the two terms in V_4 above are all *separately* positive definite.

3.3 No-go theorems in the (τ, σ)-plane

We will now restrict ourselves to the moduli τ and σ and adapt a no-go theorem from [41] that applies to all but one of the coset models to be discussed in section 4. To this end, we first isolate the contributions from σ and τ to the slow-roll parameter ϵ. To identify the contribution from σ, we use the explicit form of the inverse Kähler metric for the Kähler moduli [24]

$$K^{i\bar{j}} = 2k^i k^j - 4\mathrm{Vol}(\kappa_k k^k)^{-1\,ij}. \tag{3.17}$$

For our special intersection numbers (3.14) we then find

$$\frac{1}{4}K^{i\bar{j}}\frac{\partial V}{\partial k^i}\frac{\partial V}{\partial k^j} = \left(k^0\frac{\partial V}{\partial k^0}\right)^2 + \frac{1}{2}\left(\sigma\frac{\partial V}{\partial \sigma}\right)^2 + \left(\frac{\chi^a\chi^b}{2} - X^{-1ab}\right)\frac{\partial V}{\partial \chi^a}\frac{\partial V}{\partial \chi^b}, \tag{3.18}$$

where $\left(\frac{\chi^a\chi^b}{2} - X^{-1ab}\right)$ projects to the tangent space of the hyperplane $\chi^a X_{ab}\chi^b = 2$.

The slow-roll parameter ϵ is

$$\epsilon = \frac{K^{A\bar{B}}\partial_{\phi^A}V\partial_{\bar{\phi}^{\bar{B}}}V}{V^2} = \frac{K^{A\bar{B}}\left(\partial_{\mathrm{Re}\,\phi^A}V\partial_{\mathrm{Re}\,\phi^B}V + \partial_{\mathrm{Im}\,\phi^A}V\partial_{\mathrm{Im}\,\phi^B}V\right)}{4V^2}, \tag{3.19}$$

so that we have (using the τ-dependence of K_c spelled out in (3.8b))

$$\epsilon \geq \frac{1}{V^2}\left(\frac{1}{2}\left(\sigma\frac{\partial V}{\partial \sigma}\right)^2 + \frac{1}{4}\left(\tau\frac{\partial V}{\partial \tau}\right)^2\right) \geq \frac{1}{18V^2}\left(\sigma\frac{\partial V}{\partial \sigma} + 2\tau\frac{\partial V}{\partial \tau}\right)^2. \tag{3.20}$$

Thus, if we can show that

$$DV \equiv (-\sigma\partial_\sigma - 2\tau\partial_\tau)\,V \geq 6V, \tag{3.21}$$

we would have

$$\epsilon \geq 2\,, \quad \text{whenever } V > 0\,, \tag{3.22}$$

and slow-roll inflation and dS vacua are excluded.

From (3.13) and (3.16) we obtain, for intersection numbers of special form (3.14),

$$\begin{aligned} DV_3 &= 6V_3\,, \\ DV_{O6} &= 6V_{O6}\,, \\ DV_0 &= 6V_0\,, \\ DV_2 &= 6V_2 + \text{positive term}\,, \\ DV_4 &= 8V_4 + \text{positive term}\,, \\ DV_6 &= 10V_6\,, \end{aligned} \tag{3.23}$$

so (3.21), and hence $\epsilon \geq 2$, would follow if also $DV_f \geq 6V_f$. In [41] it was shown that the extra condition $r_{aI} = 0$ would ensure that $DV_f = 6V_f$, implying the no-go theorem. In the coset examples to be discussed in the next section, however, one has $r_{aI} \neq 0$. Therefore, we will explicitly check for each case separately whether $DV_f \geq 6V_f$ is satisfied or not. In order to do so, it is convenient to write

$$V_f = \frac{1}{2\tau^2 \mathrm{Vol}} U \,, \tag{3.24}$$

so that

$$DV_f = 6V_f + \frac{1}{2\tau^2 \mathrm{Vol}} DU = 6V_f + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma\partial_\sigma) U, \tag{3.25}$$

and the no-go theorem applies if we can show that

$$-\sigma\partial_\sigma U = -k^a \partial_{k^a} U \geq 0 \,. \tag{3.26}$$

Furthermore, if the inequality (3.26) is strictly valid, Minkowski vacua are ruled out as well. This can be seen as follows. Using (3.23) and (3.25), we obtain

$$DV = 6V + 2V_4 + 4V_6 + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma\partial_\sigma) U + \text{positive terms} \,, \tag{3.27}$$

so that for a vacuum, $DV = 0$, we find with (3.26)

$$V = -\frac{1}{6} \left(2V_4 + 4V_6 + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma\partial_\sigma) U + \text{positive terms} \right) \leq 0 \,. \tag{3.28}$$

So, if the inequality (3.26) holds strictly, also (3.28) holds strictly as well, and Minkowski vacua are ruled out.

Indeed, we checked in particular that the coset models of section 4 do not allow for *supersymmetric* Minkowski vacua with left-invariant SU(3)-structure. Strangely enough, this includes the case SU(2)×SU(2) for which eq. (3.26) can be violated. This model may still allow for a non-supersymmetric Minkowski vacuum. Incidentally, we checked also that there are no *supersymmetric* Minkowski vacua on any of the cosets of table 1 with static SU(2)-structure, which falls outside the scope of the SU(3)-structure based no-go theorem of this section.

3.4 A comment on extra ingredients

Some ingredients that are not taken into account in the original no-go theorem of [20], nor in the no-go theorems of section 3.3 are KK-monopoles, NS5-branes, D4-branes and D8-branes. Some of these ingredients were used in constructing simple dS-vacua in [31]. KK-monopoles would drastically change the topology and geometry of the internal manifold so that their introduction makes it difficult to obtain a clear ten-dimensional picture, hence we will not discuss this possibility further in this paper. NS5-branes, D4-branes and D8-branes would contribute through their respective currents j_{NS5}, j_{D4} and j_{D8} as follows to the Bianchi identities

$$\begin{aligned} \mathrm{d}H &= -j_{\mathrm{NS5}} \,, \\ \mathrm{d}F_4 + H \wedge F_2 &= -j_{\mathrm{D4}} \,, \\ \mathrm{d}F_0 &= -j_{\mathrm{D8}} \,. \end{aligned} \tag{3.29}$$

Since H and F_2 should be odd, and F_0 and F_4 even under all the orientifold involutions, we find that $j_{\rm NS5}$ is an odd four-form, $j_{\rm D4}$ an even five-form and $j_{\rm D8}$ an even one-form. In the approximation of left-invariant SU(3)-structure to be used in the next section, one should also impose these brane-currents to be left-invariant (making the branes itself smeared branes). For the concrete models of section 4 there are no such currents $j_{\rm NS5}$, $j_{\rm D4}$ or $j_{\rm D8}$ with the appropriate properties under all orientifold involutions, implying that NS5-branes, D4- and D8-branes cannot be used in these models.

Let us briefly mention that an F-term uplifting along the lines of O'KKLT [49, 50] by combining the coset models with the quantum corrected O'Raifeartaigh model will not be a promising possibility either. The O'Raifeartaigh model is given by $W_{\rm O} = -\mu^2 S$ and $K_{\rm O} = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda^2}$. The model has a dS minimum for $S = 0$ where $V_{\rm O} \approx \mu^4$. We combine the two models as follows (the subscript IIA refers to the previously discussed flux and brane contributions)

$$W = W_{\rm IIA} + W_{\rm O}\,, \qquad\qquad K = K_{\rm IIA} + K_{\rm O}\,. \tag{3.30}$$

In lowest order in S the total potential is then given by

$$V \approx V_{\rm IIA} + e^{K_{\rm IIA}} V_{\rm O} + \cdots\,. \tag{3.31}$$

Note that we can then include the contribution of $V_{\rm up} = e^{K_{\rm IIA}} V_{\rm O}$ in the no-go theorems, because the uplift potential $V_{\rm up}$ scales like F_6,

$$V_{\rm up} = \frac{A_{\rm up}}{\tau^4\,{\rm Vol}}\,. \tag{3.32}$$

Since we assume a positive uplift potential, $V_{\rm up} > 0$, the fact that $V_{\rm up}$ scales like F_6 tells us that adding this uplift potential does not help in circumventing the no-go theorems of section 2 or section 3.3.

4 Application: coset models

In the previous section, we described a no-go theorem that rules out dS vacua and slow-roll inflation for type IIA compactifications on certain types of SU(3)-structure manifolds, namely those for which one coordinate in the triple intersection number κ_{ijk} can be separated as in eq. (3.14) and the geometric fluxes induce the relation (3.26). While these seem to be quite strong assumptions, it turns out that a large part of the explicitly known examples of non-trivial SU(3)-structure compactifications actually do fall into this category, as we will show in this section.

As a starting point we could consider internal manifolds, for which an explicit SU(3)-structure compactification to a supersymmetric AdS space-time is known [40]. We are not directly interested in this AdS vacuum, but the moduli space of such a compactification might still have regions where the scalar potential is positive and allows for local dS minima or suitable inflationary trajectories. The explicitly known models with supersymmetric AdS vacua can be divided into two classes:

- Nilmanifolds (or "twisted tori")[15]
- Group manifolds and coset spaces based on semi-simple and U(1)-groups

In each case, only left-invariant SU(3)-structures are considered. The scalar curvature of a nilmanifold reads $R = -\frac{1}{4} f^{\gamma_1}{}_{\alpha_1\beta_1} f^{\gamma_2}{}_{\alpha_2\beta_2} g_{\gamma_1\gamma_2} g^{\alpha_1\alpha_2} g^{\beta_1\beta_2}$ in terms of the internal metric $g_{\alpha\beta}$ and the structure constants $f^{\gamma}{}_{\alpha\beta}$ of the nilpotent group. Apart from the torus, this is always negative so that, as discussed in section 2, the nilmanifolds provide prime candidates for avoiding the no-go theorem of [20]. However, the only known nilmanifold example, next to the torus, allowing for an $\mathcal{N} = 1$ AdS_4 solution is the Iwasawa manifold, which turns out to be T-dual to the torus solution [40, 52]. Having no geometric fluxes, the torus is ruled out by the no-go theorem of [20], and one expects this to be true then also for the Iwasawa manifold because of T-duality.

The second class of explicitly known examples, i.e. the group manifolds and coset spaces based on semi-simple and U(1)-groups, will in the following simply be referred to as "the coset models". For reviews on coset spaces see [53, 54]. As was explained in [39], in order for a coset space G/H to allow for an SU(3)-structure, the group H should be contained in SU(3).[16] The list of such six-dimensional cosets and the corresponding structure constants were given in [39] and are summarized in table 1. Out of these only five lead to $\mathcal{N} = 1$ AdS_4 solutions [39], as we have indicated in the table. In [40], the low-energy effective actions for these compactifications were calculated, so we can now check whether the no-go theorems described above can be applied.

For the cosmological applications we have in mind, however, it is not really relevant whether the effective field theories actually have supersymmetric vacua. All we are really interested in here are regions in moduli space with positive potential energy, where supersymmetry is spontaneously broken anyway. It is thus interesting to consider also compactifications that do not allow for supersymmetric AdS vacua. Still restricting to left-invariant SU(3)-structure there are only two more coset spaces of table 1. They are $\frac{\text{SU}(2)^2}{\text{U}(1)} \times \text{U}(1)$ and $\text{SU}(2)\times\text{U}(1)^3$.

In this section we will study each of these coset models separately, i.e. the seven models that allow for a left-invariant SU(3)-structure (including the five that also allow for a supersymmetric AdS vacuum). For the first four models the condition of left-invariance is very strong, and leaves only a very limited set of two-forms and three-forms as expansion forms, while there are no left-invariant one-forms nor five-forms. In these cases, we are able to show the no-go theorem without assuming that we introduce orientifolds. In the models with $G = \frac{\text{SU}(2)^2}{\text{U}(1)} \times \text{U}(1)$, $\text{SU}(2)\times\text{U}(1)^3$ and $\text{SU}(2)\times\text{SU}(2)$ there are left-invariant one-forms and five-forms, which complicates matters. For instance, the condition (3.6) becomes non-trivial. So in each of these cases, we will introduce enough orientifolds to eliminate one- and five-forms. Furthermore, we will make the simplification that the orientifolds

[15] Nilmanifolds are group manifolds of nilpotent groups, quotiented by a discrete group to make them compact. In the physics literature they are known as twisted tori, since they can be described as torus bundles on tori. See [51] for a short introduction.

[16] These coset spaces were already considered in the construction of heterotic string compactifications by [55].

G	H	$\mathcal{N}=1$ AdS$_4$
G_2	SU(3)	Yes
SU(3)×SU(2)^2	SU(3)	No
Sp(2)	S(U(2)×U(1))	Yes
SU(3)×U(1)^2	S(U(2)×U(1))	No
SU(2)^3×U(1)	S(U(2)×U(1))	No
SU(3)	U(1)×U(1)	Yes
SU(2)^2×U(1)^2	U(1)×U(1)	No
SU(3)×U(1)	SU(2)	Yes
SU(2)^3	SU(2)	No
SU(2)^2×U(1)	U(1)	No
SU(2)^2	1	Yes

Table 1. All six-dimensional manifolds of the type $M = G/H$, where H is a subgroup of SU(3) and G and H are both products of semi-simple and U(1)-groups. To be precise this list should be completed with the cosets obtained by replacing any number of SU(2) factors in G by U(1)^3.

are perpendicular to the coordinate frame, except for SU(2)×SU(2), which does not allow for perpendicular orientifolds. It turns out that in that case one can choose the same orientifolds as in the supersymmetric AdS vacua of [40] leading to the same expansion forms.

As we will show, dS vacua (as well as Minkowski) and slow-roll inflation are excluded for all these coset cases by the no-go theorem (3.26), except for the case SU(2)×SU(2).

Not requiring the presence of a supersymmetric SU(3)-structure AdS vacuum, one can consider, next to the nilmanifolds, also the solvmanifolds i.e. the group manifolds of a solvable Lie group (for related work see [33]). Without the additional conditions on the left invariance as for cosets, both nilmanifolds and solvmanifolds will however also allow for one- and five-forms, and a large amount of fields. A detailed study of all cases would probably also require considering different choices of orientifolds on each manifold, and we leave this for future work.

4.1 Models for which the no-go theorems hold

4.1.1 $\frac{G_2}{SU(3)}$

For this case, one finds for the function U of (3.24):

$$U \propto -(k^1)^2 \,, \tag{4.1}$$

which is manifestly negative. This implies that V_f itself is manifestly negative so that the no-go theorem of [20], reviewed in section 2, already rules out this case [40]. All other coset models allow for values of the moduli such that $V_f > 0$ and therefore require a more careful analysis using the refined no-go theorem of section 3.3.

4.1.2 $\frac{\mathrm{Sp}(\mathbf{2})}{\mathbf{S}(\mathrm{U}(\mathbf{2})\times\mathrm{U}(\mathbf{1}))}$

For this case, one has

$$U \propto (k^2)^2 - 4(k^1)^2 - 12k^1k^2\,, \tag{4.2}$$

and the only non-vanishing intersection number is κ_{112} and permutations thereof, so that k^2 plays the role of k^0, and we have

$$DU = -k^1\partial_{k^1}U \propto 8(k^1)^2 + 12k^1k^2 > 0\,, \tag{4.3}$$

so that with $k^i > 0$ (because of metric positivity) the inequality (3.26) is strictly satisfied and this model is ruled out.

4.1.3 $\frac{\mathrm{SU}(\mathbf{3})}{\mathrm{U}(\mathbf{1})\times\mathrm{U}(\mathbf{1})}$

For this coset space, we have

$$U \propto (k^1)^2 + (k^2)^2 + (k^3)^2 - 6k^1k^2 - 6k^2k^3 - 6k^1k^3\,, \tag{4.4}$$

and the non-vanishing intersection numbers are of the type κ_{123} so that we can choose any one of the three k's as k^0. We will choose k^0 to be the biggest and assume without loss of generality that this is k^1, i.e. that $k^1 \geq k^2, k^3$. We then find that

$$DU = (-k^2\partial_{k^2} - k^3\partial_{k^3})U \propto (6k^1 - 2k^2)k^2 + (6k^1 - 2k^3)k^3 + 12k^2k^3 > 0, \tag{4.5}$$

so that with $k^i > 0$ (because of metric positivity) this coset space is also ruled out by the no-go theorem (3.26).

4.1.4 $\frac{\mathrm{SU}(\mathbf{3})\times\mathrm{U}(\mathbf{1})}{\mathrm{SU}(\mathbf{2})}$

For this model, the function U depends on an extra constant λ related to the choice of orientifolds. The function U turns out to be

$$U \propto (k^2)^2(u^2)^2\lambda - 8k^1k^2|u^1u^2|(1+\lambda^2)\,, \tag{4.6}$$

and the non-vanishing intersection numbers are of the form κ_{112}. Thus k^2 plays the role of k^0, and we find that

$$DU = -k^1\partial_{k^1}U \propto 8k^1k^2|u^1u^2|(1+\lambda^2) > 0, \tag{4.7}$$

so that with $k^i > 0$ (because of metric positivity) this case is also ruled out.

4.1.5 $\frac{\mathrm{SU}(\mathbf{2})^2}{\mathrm{U}(\mathbf{1})} \times \mathrm{U}(\mathbf{1})$

It was shown in [39] that if the U(1) factor in the denominator does not sit completely in the $\mathrm{SU}(2)^2$, the resulting coset is equivalent to SU(2)×SU(2), so we exclude this possibility here, as the above notation already suggests. The internal manifold is then in fact equivalent to $T^{1,1}\times\mathrm{U}(1)$. We choose the structure constants as follows (this is $a = 1$, $b = 0$ compared to [39])

$$\begin{aligned} &f^1{}_{23} = f^7{}_{45} = 1, \quad \text{cyclic,} \\ &f^3{}_{45} = f^2{}_{17} = f^1{}_{72} = 1. \end{aligned} \tag{4.8}$$

The possible orientifolds that are perpendicular to the coordinate frame and compatible with these structure constants are along[17]

$$123\,,\qquad 345,\qquad 256\,,\qquad 146\,,\qquad 246\,,\qquad 156\,. \tag{4.9}$$

In order to remove one-forms and five-forms, it turns out that we have to introduce two orientifolds, in particular one of $\{123, 345\}$ and one of $\{256, 146, 246, 156\}$. It does not matter for the analysis which particular choice is made, but for definiteness let us choose 345 and 256. We arrive then at the following expansion forms

$$\begin{array}{llll} \text{odd 2-forms:} & (e^{15}+e^{24})\,, & e^{36}\,, & \\ \text{even 3-forms:} & e^{123}\,, & (e^{256}-e^{146})\,, & e^{345}\,, \end{array} \tag{4.10}$$

for (3.5).

There is always a change of basis such that we can assume $k^i > 0$. The conditions for metric positivity then become

$$u^1u^2 > 0\,,\qquad u^1u^3 > 0\,. \tag{4.11}$$

U becomes

$$U \propto \frac{-4k^1k^2u^2(u^1+u^3)+(k^2)^2\left[(u^1)^2+(u^3)^2\right]}{2\sqrt{u^1u^3}|u^2|}\,. \tag{4.12}$$

The non-vanishing intersection number is κ_{112} so that k^2 plays the role of k^0, and we get for (3.26):

$$DU = -k^1\partial_{k^1}U \propto \frac{2k^1k^2u^2(u^1+u^3)}{\sqrt{u^1u^3}|u^2|} > 0\,, \tag{4.13}$$

which is positive using the conditions (4.11). Hence, this case is ruled out as well.

4.1.6 SU(2) × U(1)^3

In this case there are ten possible orientifold planes perpendicular to the coordinate frame and compatible with the structure constants. It turns out that in order to remove the one- and five-forms we have to choose at least three mutually supersymmetric orientifolds and that it does not matter for the analysis which ones we choose. For definiteness, let us take

$$123\,,\qquad 356,\qquad 246\,. \tag{4.14}$$

With these orientifolds, we get the following expansion forms to be used in (3.5)

$$\begin{array}{lllll} \text{odd 2-forms:} & e^{16}\,, & e^{25}\,, & e^{34}\,, & \\ \text{even 3-forms:} & e^{123}\,, & e^{356}\,, & e^{264}\,, & e^{145}\,. \end{array} \tag{4.15}$$

Again there is always a change of basis such that we can assume $k^i > 0$. The positivity of the metric demands that

$$u^1u^2 > 0\,,\quad u^1u^3 > 0\,,\quad u^1u^4 > 0\,. \tag{4.16}$$

[17] To be precise e.g. 123 means for the orientifold involution $e^1 \to e^1$, $e^2 \to e^2$, $e^3 \to e^3$, $e^4 \to -e^4$, $e^5 \to -e^5$, $e^6 \to -e^6$.

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For the quantity U as defined in (3.24) we get

$$U \propto \frac{(k^1u^4)^2 + (k^2u^3)^2 + (k^3u^2)^2 - 2k^1u^4k^2u^3 - 2k^1u^4k^3u^2 - 2k^2u^3k^3u^2}{2\sqrt{u^1u^2u^3u^4}} \,. \tag{4.17}$$

The non-vanishing intersection number is κ_{123} so that each k^i can play the role of k^0. Without loss of generality we can assume $k^1u^4 \geq k^2u^3 > 0$, $k^1u^4 \geq k^3u^2 > 0$ and choose k^0 to be k^1. Thus we then find

$$DU = (-k^2\partial_{k^2} - k^3\partial_{k^3})U \propto \frac{-(k^2u^3 - k^3u^2)^2 + k^1u^4(k^2u^3 + k^3u^2)}{\sqrt{u^1u^2u^3u^4}} > 0 \,, \tag{4.18}$$

so that we can also rule out this model.

4.2 SU(2) × SU(2)

Thus far, we have found that $\epsilon \geq 2$ for all other cases. For the remaining coset space SU(2)×SU(2), one finds

$$\begin{aligned} U \propto & \sum_{i=1}^{3}(k^i)^2\left(\sum_{I=1}^{4}(u^I)^2\right) - 4k^2k^3(|u^1u^2| + |u^3u^4|) \\ & - 4k^1k^2(|u^1u^4| + |u^2u^3|) - 4k^1k^3\left(|u^1u^3| + |u^2u^4|\right) \,, \end{aligned} \tag{4.19}$$

and the non-vanishing intersection numbers are of the form κ_{123} so that we could choose any one of the k's as k^0. However, it is not possible to apply the no-go theorem. This can be easily seen if we take for example $u^1 \gg u^2, u^3, u^4$. Then we have schematically $U \propto \vec{k}^2(u^1)^2$ and $DU \propto -k^ak^a(u^1)^2 < 0$. In [41] further no-go theorems have been derived but none of those apply to this case either. Let us therefore study it in more detail.

4.2.1 Small ϵ for SU(2) × SU(2)

We have argued above that the known no-go theorems cannot be used to rule out small ϵ for this compactification. Indeed we will see that $\epsilon \approx 0$ is possible and there are dS extrema.

The superpotential and Kähler potential of the effective $\mathcal{N} = 1$ supergravity have been derived in various ways in [44–46] (based on earlier work of [23, 24, 56]). Here we summarize the main formulæ which will be used in the following. The superpotential for SU(3)-structure reads in the Einstein frame

$$\mathcal{W} = \frac{1}{4\kappa_{10}^2}\int_M \langle e^{i(J-i\delta B)}, F - i\mathrm{d}_H\left(e^{-\Phi}\mathrm{Im}\,\Omega + i\delta C_3\right)\rangle \,, \tag{4.20}$$

where $\langle\phi_1, \phi_2\rangle = \phi_1 \wedge \lambda(\phi_2)|_{\mathrm{top}}$ is the Mukai pairing. λ is the operator reversing the indices of a form and $|_{\mathrm{top}}$ selects the part of top dimension six, as necessary to integrate over the internal manifold M. The scalar potential is given in terms of the superpotential via

$$V(\phi, \bar{\phi}) = M_p^{-2}e^{\mathcal{K}}\left(\mathcal{K}^{A\bar{B}}D_A\mathcal{W}D_{\bar{B}}\mathcal{W}^* - 3|\mathcal{W}|^2\right) . \tag{4.21}$$

In order to eliminate the one- and five-forms we must introduce at least three mutually supersymmetric orientifolds, compatible with the structure constants. We can then always perform a basis transformation so that the odd two-forms and odd/even three-forms are the same as in [40] and read[18]

$$\begin{aligned}
Y_1^{(2-)} =& e^{14}, \qquad Y_2^{(2-)} = e^{25}, \qquad Y_3^{(2-)} = e^{36},\\
Y^{(3-)1} =& \frac{1}{4}\left(e^{156} - e^{234} - e^{246} + e^{135} + e^{345} - e^{126} + e^{123} - e^{456}\right),\\
Y^{(3-)2} =& \frac{1}{4}\left(e^{156} - e^{234} + e^{246} - e^{135} - e^{345} + e^{126} + e^{123} - e^{456}\right),\\
Y^{(3-)3} =& \frac{1}{4}\left(e^{156} - e^{234} + e^{246} - e^{135} + e^{345} - e^{126} - e^{123} + e^{456}\right),\\
Y^{(3-)4} =& \frac{1}{4}\left(-e^{156} + e^{234} + e^{246} - e^{135} + e^{345} - e^{126} + e^{123} - e^{456}\right),\\
Y_1^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} - e^{246} - e^{135} + e^{345} + e^{126} + e^{123} + e^{456}\right),\\
Y_2^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} + e^{246} + e^{135} - e^{345} - e^{126} + e^{123} + e^{456}\right),\\
Y_3^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} + e^{246} + e^{135} + e^{345} + e^{126} - e^{123} - e^{456}\right),\\
Y_4^{(3+)} =& \frac{1}{2}\left(-e^{156} - e^{234} + e^{246} + e^{135} + e^{345} + e^{126} + e^{123} + e^{456}\right),
\end{aligned} \tag{4.22}$$

where the e^α $(\alpha = 1, \ldots, 6)$ are a basis of left-invariant 1-forms, and we use the shorthand notation $e^{14} = e^1 \wedge e^4$ etc. The e^α satisfy

$$\mathrm{d}e^\alpha = -\frac{1}{2} f^\alpha{}_{\beta\gamma} e^\beta \wedge e^\gamma\,, \tag{4.23}$$

where the structure constants for $\mathrm{SU}(2) \times \mathrm{SU}(2)$ are $f^1{}_{23} = f^4{}_{56} = 1$, cyclic.[19] From this we find

$$\mathrm{d}Y_i^{(2-)} = r_{iI} Y^{(3-)I}, \quad \text{with} \quad r = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \end{pmatrix}. \tag{4.24}$$

In terms of the above expansion forms, we can again define the complex moduli as in (3.5). The positivity of the metric demands

$$u^1 u^2 < 0\,, \quad u^3 u^4 < 0\,, \quad u^1 u^4 < 0\,. \tag{4.25}$$

Next we turn to the choice of background fluxes. As explained in appendix A, for the part of the moduli space where H is non-trivial in cohomology, $p \neq 0$ (see below), the most general form of the background fluxes is

$$F_0 = m, \tag{4.26a}$$

$$F_2 = m^i Y_i^{(2-)}, \tag{4.26b}$$

[18] There are no even two-forms for our choice of orientifold involutions.

[19] This model can be thought of as a twisted version of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ as discussed in [37]. In that paper the authors focused on moduli stabilization and model building while we are interested in the cosmological aspects.

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$$F_4 = 0, \tag{4.26c}$$
$$F_6 = 0, \tag{4.26d}$$
$$H = p\left(Y_1^{(3-)} + Y_2^{(3-)} - Y_3^{(3-)} + Y_4^{(3-)}\right). \tag{4.26e}$$

Plugging in these background values for the fluxes together with the expansion (3.5) in terms of the basis (4.22), we find for the superpotential (4.20)

$$\mathcal{W} = V_s(4\kappa_{10}^2)^{-1}\left(m^1t^2t^3 + m^2t^1t^3 + m^3t^1t^2 - imt^1t^2t^3 - ip(z^1 + z^2 - z^3 + z^4) + r_{iI}t^iz^I\right), \tag{4.27}$$

and the Kähler potential

$$K = -\ln\prod_{i=1}^{3}\left(t^i + \bar{t}^i\right) - \ln\prod_{I=1}^{4}\left(z^I + \bar{z}^I\right) + 3\ln\left(V_s^{-1}\kappa_{10}^2M_P^2\right) + \ln 32\,, \tag{4.28}$$

where $V_s = -\int_M e^{123456}$. Note that the superpotential depends on all the moduli so there are no flat directions in this model.

It is straightforward to calculate the scalar potential (4.21) and the slow-roll parameter ϵ from the Kähler and superpotential. Although we cannot analytically minimize ϵ, we can do it numerically. One particular solution with numerically vanishing ϵ is

$$\begin{aligned}
&m^1 = m^2 = m^3 = L\,, && m = 2\,L^{-1}\,, \qquad p = 3\,L^2\,,\\
&k^1 = k^2 = k^3 \approx .8974\,L^2\,, && b^1 = b^2 = b^3 \approx -.8167\,L^2\,,\\
&u^1 \approx 2.496\,L^3, && u^2 = -u^3 = u^4 \approx -.05667\,L^3\,,\\
&c^1 \approx -2.574\,L^3\,, && c^2 = -c^3 = c^4 \approx .3935\,L^3\,,
\end{aligned} \tag{4.29}$$

where L is an arbitrary length. While we can use L to scale up our solution with respect to the string length l_s, we stress that this does not correspond to a massless modulus, as it also changes the fluxes.

We conclude that in this case there is no lower bound for ϵ. To obtain a trustworthy supergravity solution we would have to make sure that the internal space is large compared to the string length and that the string coupling is small (for which we could use our freedom in L). Furthermore, in the full string theory the fluxes have to be properly quantized, implying in particular some quantization on L, and tadpole cancellation conditions have to be satisfied. Although we do not think that this would necessarily prevent small ϵ, we did not try to find a solution with all these constraints satisfied, because all the solutions with vanishing epsilon we found have a more serious problem, namely that $\eta \lesssim -2.4$.

The eigenvalues of the mass matrix turn out to be generically all positive except for one, with the one tachyonic direction being a mixture of all the light fields, in particular the axions. This means that we have a saddle point rather than a dS minimum. A similar instability was found in related models in [41].

In [42], a no-go theorem preventing dS vacua and slow-roll inflation was derived by studying the eigenvalues of the mass matrix. Allowing for an arbitrary tuning of the superpotential it was shown that for certain Kähler potentials the Goldstino mass is always

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negative. For the examples we found, this mass is always positive so that the no-go theorem of [42] does not apply. This means that allowing for an arbitrary superpotential it should be possible to remove the tachyonic direction. In our case, however, the superpotential is of course not arbitrary.

Since the no-go theorems against slow-roll inflation do not apply and we have found solutions with vanishing ϵ, we checked whether our solutions allow for small η in the vicinity of the dS extrema. Unfortunately, this is not the case. In fact, we found that η does not change much in the vicinity of our solutions where ϵ is still small.

It would be very interesting to study the SU(2)×SU(2) model further to check whether one can prove that there is always at least one tachyonic direction or whether it allows for metastable dS vacua after all. Understanding this tachyonic direction better should also allow to decide whether there are points in the moduli space that allow for slow-roll inflation in this model.

5 Conclusions and outlook

Type IIA compactifications on orientifolds of SU(3)-structure manifolds with fluxes and D6-branes are phenomenologically interesting because they lead to effective 4D $\mathcal{N} = 1$ supergravity actions with rich potentials for the moduli. These potentials have a dilaton-volume dependence that forbids dS vacua or slow-roll inflation unless the compact space has a negative scalar curvature induced by the geometric fluxes (or other more complex ingredients are introduced [20, 31, 33]).

Motivated by this, we analysed a class of explicitly known SU(3)-structure compactifications with fluxes and O6/D6-sources for which the full scalar potential can be written down in closed form. The manifolds we studied are those coset spaces or group manifolds based on semi-simple and U(1) groups that admit a left-invariant SU(3)-structure [39].

As indicated in table 1, five out of these seven manifolds allow for 4D $\mathcal{N} = 1$ AdS solutions that solve the full 10D field equations of massive IIA supergravity [39]. Apart from a particular nilmanifold (the Iwasawa manifold) and tori, these are, to the authors best knowledge, the only explicitly known examples of this type. Using the 4D effective action worked out in [40], we could rule out dS (as well as Minkowski) vacua and slow-roll inflation elsewhere in moduli space for four of these coset spaces by using a refined no-go theorem that probes the scalar potential also along a Kähler modulus different from the overall volume modulus (see also [41]). Just as the no-go theorem of [20], this no-go theorem works by establishing a certain lower bound on the first derivatives of the potential, and hence the epsilon parameter, for $V \geq 0$. It is thus different in spirit from the no-go theorems given in [42], which assume a small first derivative and consider consequences for the second derivatives, i.e. the eta parameter.

The only coset space that allows for supersymmetric vacua and that is not directly ruled out by any known no-go theorem is then the group manifold SU(2)×SU(2). For this case, we were indeed able to find critical points (corresponding to numerically vanishing ϵ) with positive energy density, but only at the price of a tachyonic direction, corresponding to a large negative eta-parameter, $\eta \lesssim -2.4$. Interestingly, this tachyonic direction does not correspond to the one used in the different types of no-go theorems of [42]. As our numerical search was not exhaustive, however, we cannot completely rule out the existence of dS vacua or inflating regions for this case. Since this case also does not allow for a supersymmetric Minkowski vacuum as mentioned below (3.28), our discussion covers all SU(3)-structure compactifications on semi-simple and U(1) cosets that have a supersymmetric vacuum.

Furthermore, we also studied the remaining two coset spaces of table 1 that do admit an SU(3)-structure but no supersymmetric AdS vacuum. Choosing for simplicity the O-planes such that one-forms are projected out and restricting to O-planes perpendicular to the coordinate frame, we could again use the refined no-go theorem of section 3.3 to rule out dS vacua and slow-roll inflation for both of these cases as well.

Our results show that a negative scalar curvature and a non-vanishing F_0 is in general not enough to ensure dS vacua or inflation (as also noted in [33]), and we give a geometric criterion that allows one to separate interesting SU(3)-structure compactifications from non-realistic ones.

Our study could be extended in several directions. For one thing, it would be extremely interesting to find explicit SU(3)-structure manifolds that do not fall under the class of coset spaces we have discussed here and to investigate their usefulness for cosmological applications along the lines of this paper. The most obvious class of manifolds to study systematically would be the nil- and solvmanifolds. Another interesting direction might be the study of compactifications on manifolds with $\mathcal{N} = 1$ spinor ansätze more general than the SU(3)-structure case [57]. Concerning the SU(2)$\times$SU(2) model discussed in our paper, one might try to either find a working dS minimum, or rule it out based on another no-go theorem, perhaps by using methods similar in spirit to [42], although a direct application of their results to this case does not seem possible. Following [31, 32] or [58–60], one could also try to incorporate additional structures such as NS5-branes or quantum corrections of various types. In section 3.4, however, we found that at least for our models, the following additional ingredients cannot be added or do not work: NS5-, D4- and D8-branes as well as an F-term uplift along the lines of O'KKLT [49, 50]. Perhaps also methods similar to the ones in [61] for non-supersymmetric Minkowski or AdS vacua might be useful for the direct 10D construction of dS compactifications. There is certainly a lot to improve about our understanding of cosmologically realistic compactifications of the type IIA string!

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A Labelling the disconnected bubbles of moduli space by flux quanta

In section 4.2 we will search for a configuration with small ϵ somewhere in the moduli space. As we will argue in a moment, this moduli space consists of different disconnected "bubbles", i.e. these bubbles are such that it is not possible to reach another bubble by finite fluctuations of the moduli fields. The approach of [40] of starting from a supersymmetric configuration and expanding around it is inadequate for studying the whole configuration space since on the one hand, there will be bubbles that do not contain a supersymmetric configuration, while on the other hand, there are bubbles that contain more than one supersymmetric configuration. In fact, in section 4.2 we find configurations with $\epsilon \approx 0$ and $V > 0$ in bubbles that do not allow for supersymmetric AdS vacua. We follow here the standard approach of classifying the moduli space by flux quanta, which is however complicated by the presence of Romans mass, H-field and O6-plane source.

Classifying the different bubbles in terms of fluxes amounts to finding configurations that solve the Bianchi identities

$$\mathrm{d}H = 0\,, \tag{A.1a}$$
$$\mathrm{d}F_0 = 0\,, \tag{A.1b}$$
$$\mathrm{d}F_2 + mH = -j_3\,, \tag{A.1c}$$
$$\mathrm{d}F_4 + H \wedge F_2 = 0\,, \tag{A.1d}$$

while two configurations are considered equivalent if they are related by a fluctuation of the moduli fields, which after imposing the orientifold projection (and assuming it removes one-forms) is given by [40]

$$\delta H = \mathrm{d}\delta B\,, \tag{A.2a}$$
$$\delta F_0 = 0\,, \tag{A.2b}$$
$$\delta F_2 = -m\delta B\,, \tag{A.2c}$$
$$\delta F_4 = \mathrm{d}\delta C_3 - \delta B \wedge (F_2 + \delta F_2) - \frac{1}{2}m(\delta B)^2\,, \tag{A.2d}$$
$$\delta F_6 = H \wedge \delta C_3 - \delta B \wedge (F_4 + \delta F_4) - \frac{1}{2}(\delta B)^2 \wedge (F_2 + \delta F_2) - \frac{1}{3!}m(\delta B)^3\,. \tag{A.2e}$$

In other words, we want to find representatives of the cohomology of the Bianchi identities (A.1) modulo pure fluctuations of the potentials (A.2).

From eqs. (A.1a), (A.1b), (A.2a) and (A.2b) follows immediately that $H \in H^3(M,\mathbb{R})$ and F_0 constant. To analyse (A.1c) and (A.2c) we take the point of view that we choose the flux F_2, which then determines the source j_3. In fact, if $F_0 \neq 0$ the flux F_2 is only determined up to a closed form, since the fluctuation δB was from (A.2a) also only determined up to a closed form, which can then be used in (A.2c). Moving on to F_4, we find that in eq. (A.1d) $H \wedge F_2 = 0$, since we assumed there were no even five-forms under all the orientifold involutions. Moreover, with the fluctuations δC_3 we can remove the exact part of F_4 so that $F_4 \in H^4(M,\mathbb{R})$. This however, leaves the closed part of δC_3 undetermined, which, if we have chosen H non-trivial, can in the SU(2)×SU(2) case be used to put $F_6 = 0$.

Taking into account the parity requirements under the orientifold involution, we find for the case of SU(2)×SU(2) the general form of the background eq. (4.26) when H is non-trivial. If H is trivial one must allow for non-zero F_6.

References

[1] WMAP collaboration, E. Komatsu et al., *Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Cosmological Interpretation*, *Astrophys. J. Suppl.* **180** (2009) 330 [arXiv:0803.0547] [SPIRES];
WMAP collaboration, D.N. Spergel et al., *Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology*, *Astrophys. J. Suppl.* **170** (2007) 377 [astro-ph/0603449] [SPIRES].

[2] A.H. Guth, *The inflationary universe: a possible solution to the horizon and flatness problems*, *Phys. Rev.* **D 23** (1981) 347 [SPIRES];
A.D. Linde, *A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, *Phys. Lett.* **B 108** (1982) 389 [SPIRES];
A.J. Albrecht and P.J. Steinhardt, *Cosmology for grand unified theories with radiatively induced symmetry breaking*, *Phys. Rev. Lett.* **48** (1982) 1220 [SPIRES].

[3] V. Mukhanov, *Physical foundations of cosmology*, Cambridge University Press, Cambridge U.K. (2005);
S. Weinberg, *Cosmology*, Oxford University Press, U.S.A. (2008);
A.D. Linde, *Particle physics and inflationary cosmology*, Haarwood, Chur Switzerland (1990) [hep-th/0503203] [SPIRES];
A.R. Liddle and D.H. Lyth, *Cosmological inflation and large-scale structure*, Cambridge University Press, Cambridge U.K. (2000).

[4] Supernova Search Team collaboration, A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201] [SPIRES];
Supernova Cosmology Project collaboration, S. Perlmutter et al., *Measurements of* Ω *and* Λ *from* 42 *high-redshift supernovae*, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133] [SPIRES].

[5] S.H. Henry Tye, *Brane inflation: string theory viewed from the cosmos*, *Lect. Notes Phys.* **737** (2008) 949 [hep-th/0610221] [SPIRES];
J.M. Cline, *String cosmology*, hep-th/0612129 [SPIRES];
R. Kallosh, *On inflation in string theory*, *Lect. Notes Phys.* **738** (2008) 119 [hep-th/0702059] [SPIRES];
A. Linde, *Inflationary cosmology*, *Lect. Notes Phys.* **738** (2008) 1 [arXiv:0705.0164] [SPIRES];
C.P. Burgess, *Lectures on Cosmic Inflation and its Potential Stringy Realizations*, PoS(P2GC)008 *Class. Quant. Grav.* **24** (2007) S795 [arXiv:0708.2865] [SPIRES];
L. McAllister and E. Silverstein, *String cosmology: a review*, *Gen. Rel. Grav.* **40** (2008) 565 [arXiv:0710.2951] [SPIRES].

[6] C. Herdeiro, S. Hirano and R. Kallosh, *String theory and hybrid inflation/acceleration*, *JHEP* **12** (2001) 027 [hep-th/0110271] [SPIRES];

K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, *D3/D7 inflationary model and M-theory*, *Phys. Rev.* **D 65** (2002) 126002 [hep-th/0203019] [SPIRES];
K. Dasgupta, J.P. Hsu, R. Kallosh, A. Linde and M. Zagermann, *D3/D7 brane inflation and semilocal strings*, *JHEP* **08** (2004) 030 [hep-th/0405247] [SPIRES];
M. Haack et al., *Update of D3/D7-brane inflation on* $K3 \times T^2/\mathbb{Z}_2$, *Nucl. Phys.* **B 806** (2009) 103 [arXiv:0804.3961] [SPIRES];
C.P. Burgess, J.M. Cline and M. Postma, *Axionic* $D3-D7$ *inflation*, arXiv:0811.1503 [SPIRES].

[7] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240] [SPIRES].

[8] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055] [SPIRES].

[9] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097] [SPIRES].

[10] R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional string compactifications with D-branes, orientifolds and fluxes*, *Phys. Rept.* **445** (2007) 1 [hep-th/0610327] [SPIRES].

[11] C. Angelantonj and A. Sagnotti, *Open strings*, *Phys. Rept.* **371** (2002) 1 [*Erratum ibid.* **376** (2003) 339] [hep-th/0204089] [SPIRES];
A.M. Uranga, *Chiral four-dimensional string compactifications with intersecting D-branes*, *Class. Quant. Grav.* **20** (2003) S373 [hep-th/0301032] [SPIRES];
E. Kiritsis, *D-branes in standard model building, gravity and cosmology*, *Fortsch. Phys.* **52** (2004) 200 *Phys. Rept.* **421** (2005) 105 [*Erratum ibid.* **429** (2006) 121] [hep-th/0310001] [SPIRES];
F.G. Marchesano Buznego, *Intersecting D-brane models*, hep-th/0307252 [SPIRES];
D. Lüst, *Intersecting brane worlds: a path to the standard model?*, *Class. Quant. Grav.* **21** (2004) S1399 [hep-th/0401156] [SPIRES];
R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71 [hep-th/0502005] [SPIRES].

[12] F. Gmeiner and G. Honecker, *Millions of standard models on* $\mathbb{Z}_6'$*?*, *JHEP* **07** (2008) 052 [arXiv:0806.3039] [SPIRES].

[13] R. Blumenhagen, V. Braun, T.W. Grimm and T. Weigand, *GUTs in type IIB orientifold compactifications*, arXiv:0811.2936 [SPIRES].

[14] R. Donagi and M. Wijnholt, *Model building with F-theory*, arXiv:0802.2969 [SPIRES];
C. Beasley, J.J. Heckman and C. Vafa, *GUTs and exceptional branes in F-theory — I*, arXiv:0802.3391 [SPIRES].

[15] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [hep-th/0505160] [SPIRES].

[16] J.-P. Derendinger, C. Kounnas, P.M. Petropoulos and F. Zwirner, *Superpotentials in IIA compactifications with general fluxes*, *Nucl. Phys.* **B 715** (2005) 211 [hep-th/0411276] [SPIRES].

[17] P.G. Cámara, A. Font and L.E. Ibáñez, *Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold*, *JHEP* **09** (2005) 013 [hep-th/0506066] [SPIRES].

[18] G. Villadoro and F. Zwirner, $\mathcal{N}=1$ *effective potential from dual type- IIA D6/O6*

orientifolds with general fluxes, JHEP **06** (2005) 047 [hep-th/0503169] [SPIRES].

[19] M.P. Hertzberg, M. Tegmark, S. Kachru, J. Shelton and O. Ozcan, *Searching for inflation in simple string theory models: an astrophysical perspective*, Phys. Rev. **D 76** (2007) 103521 [arXiv:0709.0002] [SPIRES].

[20] M.P. Hertzberg, S. Kachru, W. Taylor and M. Tegmark, *Inflationary constraints on type IIA string theory*, JHEP **12** (2007) 095 [arXiv:0711.2512] [SPIRES].

[21] S. Chiossi and S. Salamon, *The intrinsic torsion of* SU(3)- *and* G_2*-structures*, in *Differential Geometry, Valencia Spain* 2001, World Scientific Publishing, Singapore (2002), pg. 115 [math.DG/0202282] [SPIRES].

[22] S. Gurrieri, J. Louis, A. Micu and D. Waldram, *Mirror symmetry in generalized Calabi-Yau compactifications*, Nucl. Phys. **B 654** (2003) 61 [hep-th/0211102] [SPIRES];
G. Lopes Cardoso et al., *Non-Kähler string backgrounds and their five torsion classes*, Nucl. Phys. **B 652** (2003) 5 [hep-th/0211118] [SPIRES].

[23] T.W. Grimm and J. Louis, *The effective action of* $N = 1$ *Calabi-Yau orientifolds*, Nucl. Phys. **B 699** (2004) 387 [hep-th/0403067] [SPIRES].

[24] T.W. Grimm and J. Louis, *The effective action of type IIA Calabi-Yau orientifolds*, Nucl. Phys. **B 718** (2005) 153 [hep-th/0412277] [SPIRES].

[25] J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, Nucl. Phys. **B 635** (2002) 395 [hep-th/0202168] [SPIRES];
S. Fidanza, R. Minasian and A. Tomasiello, *Mirror symmetric* SU(3)*-structure manifolds with NS fluxes*, Commun. Math. Phys. **254** (2005) 401 [hep-th/0311122] [SPIRES];
M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, JHEP **08** (2004) 046 [hep-th/0406137] [SPIRES];
R. D'Auria, S. Ferrara, M. Trigiante and S. Vaula, *Gauging the Heisenberg algebra of special quaternionic manifolds*, Phys. Lett. **B 610** (2005) 147 [hep-th/0410290] [SPIRES];
A. Micu, E. Palti and G. Tasinato, *Towards Minkowski vacua in type II string compactifications*, JHEP **03** (2007) 104 [hep-th/0701173] [SPIRES];
D. Cassani and A. Bilal, *Effective actions and* $N = 1$ *vacuum conditions from* $\mathrm{SU}(3) \times \mathrm{SU}(3)$ *compactifications*, JHEP **09** (2007) 076 [arXiv:0707.3125] [SPIRES];
G. Villadoro and F. Zwirner, *On general flux backgrounds with localized sources*, JHEP **11** (2007) 082 [arXiv:0710.2551] [SPIRES];
I. Benmachiche, J. Louis and D. Martínez-Pedrera, *The effective action of the heterotic string compactified on manifolds with* SU(3) *structure*, Class. Quant. Grav. **25** (2008) 135006 [arXiv:0802.0410] [SPIRES].

[26] A.-K. Kashani-Poor and R. Minasian, *Towards reduction of type-II theories on* SU(3) *structure manifolds*, JHEP **03** (2007) 109 [hep-th/0611106] [SPIRES];
A.-K. Kashani-Poor, *Nearly Kähler reduction*, JHEP **11** (2007) 026 [arXiv:0709.4482] [SPIRES].

[27] M. Ihl, D. Robbins and T. Wrase, *Toroidal orientifolds in IIA with general NS-NS fluxes*, JHEP **08** (2007) 043 [arXiv:0705.3410] [SPIRES].

[28] D. Robbins and T. Wrase, *D-terms from generalized NS-NS fluxes in type II*, JHEP **12** (2007) 058 [arXiv:0709.2186] [SPIRES].

[29] M. Ihl and T. Wrase, *Towards a realistic type IIA* $T^6/\mathbb{Z}_4$ *orientifold model with background fluxes. I: moduli stabilization*, JHEP **07** (2006) 027 [hep-th/0604087] [SPIRES].

[30] M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003] [SPIRES];
M.R. Douglas and S. Kachru, *Flux compactification*, *Rev. Mod. Phys.* **79** (2007) 733 [hep-th/0610102] [SPIRES];
F. Denef, *Les Houches lectures on constructing string vacua*, arXiv:0803.1194 [SPIRES].

[31] E. Silverstein, *Simple de Sitter solutions*, *Phys. Rev.* **D 77** (2008) 106006 [arXiv:0712.1196] [SPIRES].

[32] E. Silverstein and A. Westphal, *Monodromy in the CMB: gravity waves and string inflation*, *Phys. Rev.* **D 78** (2008) 106003 [arXiv:0803.3085] [SPIRES];
L. McAllister, E. Silverstein and A. Westphal, *Gravity waves and linear inflation from axion monodromy*, arXiv:0808.0706 [SPIRES].

[33] S.S. Haque, G. Shiu, B. Underwood and T. Van Riet, *Minimal simple de Sitter solutions*, arXiv:0810.5328 [SPIRES].

[34] K. Behrndt and M. Cvetič, *General $\mathcal{N}=1$ supersymmetric flux vacua of (massive) type IIA string theory*, *Phys. Rev. Lett.* **95** (2005) 021601 [hep-th/0403049] [SPIRES]; *General $\mathcal{N}=1$ supersymmetric fluxes in massive type IIA string theory*, *Nucl. Phys.* **B 708** (2005) 45 [hep-th/0407263] [SPIRES].

[35] D. Lüst and D. Tsimpis, *Supersymmetric AdS_4 compactifications of IIA supergravity*, *JHEP* **02** (2005) 027 [hep-th/0412250] [SPIRES].

[36] T. House and E. Palti, *Effective action of (massive) IIA on manifolds with* SU(3) *structure*, *Phys. Rev.* **D 72** (2005) 026004 [hep-th/0505177] [SPIRES].

[37] G. Aldazabal and A. Font, *A second look at $N=1$ supersymmetric AdS_4 vacua of type IIA supergravity*, *JHEP* **02** (2008) 086 [arXiv:0712.1021] [SPIRES].

[38] A. Tomasiello, *New string vacua from twistor spaces*, *Phys. Rev.* **D 78** (2008) 046007 [arXiv:0712.1396] [SPIRES].

[39] P. Koerber, D. Lüst and D. Tsimpis, *Type IIA AdS_4 compactifications on cosets, interpolations and domain walls*, *JHEP* **07** (2008) 017 [arXiv:0804.0614] [SPIRES].

[40] C. Caviezel et al., *The effective theory of type IIA AdS_4 compactifications on nilmanifolds and cosets*, *Class. Quant. Grav.* **26** (2009) 025014 [arXiv:0806.3458] [SPIRES];
S. Körs, *On the effective theory of type IIA AdS_4 compactifications*, arXiv:0810.5104 [SPIRES].

[41] R. Flauger, S. Paban, D. Robbins and T. Wrase, *On slow-roll moduli inflation in massive IIA supergravity with metric fluxes*, arXiv:0812.3886 [SPIRES].

[42] R. Brustein, S.P. de Alwis and E.G. Novak, *M-theory moduli space and cosmology*, *Phys. Rev.* **D 68** (2003) 043507 [hep-th/0212344] [SPIRES];
I. Ben-Dayan, R. Brustein and S.P. de Alwis, *Models of modular inflation and their phenomenological consequences*, *JCAP* **07** (2008) 011 [arXiv:0802.3160] [SPIRES];
M. Badziak and M. Olechowski, *Volume modulus inflation and a low scale of SUSY breaking*, *JCAP* **07** (2008) 021 [arXiv:0802.1014] [SPIRES];
L. Covi et al., *De Sitter vacua in no-scale supergravities and Calabi-Yau string models*, *JHEP* **06** (2008) 057 [arXiv:0804.1073] [SPIRES]; *Constraints on modular inflation in supergravity and string theory*, *JHEP* **08** (2008) 055 [arXiv:0805.3290] [SPIRES];
M. Gomez-Reino, J. Louis and C.A. Scrucca, *No metastable de Sitter vacua in $N=2$ supergravity with only hypermultiplets*, *JHEP* **02** (2009) 003 [arXiv:0812.0884] [SPIRES].

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[43] E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *New formulations of $D = 10$ supersymmetry and D8-O8 domain walls*, *Class. Quant. Grav.* **18** (2001) 3359 [hep-th/0103233] [SPIRES].

[44] M. Graña, J. Louis and D. Waldram, *Hitchin functionals in $N = 2$ supergravity*, *JHEP* **01** (2006) 008 [hep-th/0505264] [SPIRES]; SU(3) × SU(3) *compactification and mirror duals of magnetic fluxes*, *JHEP* **04** (2007) 101 [hep-th/0612237] [SPIRES].

[45] I. Benmachiche and T.W. Grimm, *Generalized $N = 1$ orientifold compactifications and the Hitchin functionals*, *Nucl. Phys.* **B 748** (2006) 200 [hep-th/0602241] [SPIRES].

[46] P. Koerber and L. Martucci, *From ten to four and back again: how to generalize the geometry*, *JHEP* **08** (2007) 059 [arXiv:0707.1038] [SPIRES].

[47] N.J. Hitchin, *The geometry of three-forms in six and seven dimensions*, math.DG/0010054 [SPIRES].

[48] D. Cassani, *Reducing democratic type-II supergravity on* SU(3) × SU(3) *structures*, *JHEP* **06** (2008) 027 [arXiv:0804.0595] [SPIRES].

[49] R. Kallosh and A. Linde, *O'KKLT*, *JHEP* **02** (2007) 002 [hep-th/0611183] [SPIRES].

[50] R. Kallosh and M. Soroush, *Issues in type IIA uplifting*, *JHEP* **06** (2007) 041 [hep-th/0612057] [SPIRES].

[51] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *A scan for new $N = 1$ vacua on twisted tori*, *JHEP* **05** (2007) 031 [hep-th/0609124] [SPIRES].

[52] C. Kounnas, D. Lüst, P.M. Petropoulos and D. Tsimpis, *AdS_4 flux vacua in type-II superstrings and their domain-wall solutions*, *JHEP* **09** (2007) 051 [arXiv:0707.4270] [SPIRES].

[53] P. van Nieuwenhuizen, *General theory of coset manifolds and antisymmetric tensors applied to Kaluza-Klein supergravity*, in *Supersymmetry and supergravity '*84*, Proceedings of the Trieste Spring School Trieste Italy April* 4–14 1984, World Scientific, Singapore (1985);
D. Kapetanakis and G. Zoupanos, *Coset space dimensional reduction of gauge theories*, *Phys. Rept.* **219** (1992) 4 [SPIRES].

[54] F. Müller-Hoissen and R. Stückl, *Coset spaces and ten-dimensional unified theories*, *Class. Quant. Grav.* **5** (1988) 27 [SPIRES].

[55] D. Lüst, *Compactification of ten-dimensional superstring theories over Ricci flat coset spaces*, *Nucl. Phys.* **B 276** (1986) 220 [SPIRES];
L. Castellani and D. Lüst, *Superstring compactification on homogeneous coset spaces with torsion*, *Nucl. Phys.* **B 296** (1988) 143 [SPIRES].

[56] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [*Erratum ibid.* **B 608** (2001) 477] [hep-th/9906070] [SPIRES];
S. Gukov, *Solitons, superpotentials and calibrations*, *Nucl. Phys.* **B 574** (2000) 169 [hep-th/9911011] [SPIRES];
T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152] [SPIRES].

[57] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Generalized structures of $N = 1$ vacua*, *JHEP* **11** (2005) 020 [hep-th/0505212] [SPIRES].

[58] F. Saueressig, U. Theis and S. Vandoren, *On de Sitter vacua in type IIA orientifold compactifications*, *Phys. Lett.* **B 633** (2006) 125 [hep-th/0506181] [SPIRES].

[59] M. Davidse, F. Saueressig, U. Theis and S. Vandoren, *Membrane instantons and de Sitter vacua*, *JHEP* **09** (2005) 065 [hep-th/0506097] [SPIRES].

[60] E. Palti, G. Tasinato and J. Ward, *WEAKLY-coupled IIA flux compactifications*, *JHEP* **06** (2008) 084 [arXiv:0804.1248] [SPIRES].

[61] D. Lüst, F. Marchesano, L. Martucci and D. Tsimpis, *Generalized non-supersymmetric flux vacua*, *JHEP* **11** (2008) 021 [arXiv:0807.4540] [SPIRES].

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